

New U(1) Gauge Model of Radiative Lepton Masses with Sterile Neutrino and Dark Matter

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Abstract

An anomaly-free U(1) gauge extension of the standard model (SM) is presented. Only one Higgs doublet with a nonzero vacuum expectation is required as in the SM. New fermions and scalars as well as all SM particles transform nontrivially under this U(1), resulting in a model of three active neutrinos and one sterile neutrino, all acquiring radiative masses. Charged-lepton masses are also radiative as well as the mixing between active and sterile neutrinos. At the same time, a residual Z_2 symmetry of the U(1) gauge symmetry remains exact, allowing for the existence of dark matter.

The notion that neutrino mass is connected to dark matter has motivated a large number of studies in recent years. The simplest realization is the one-loop “scotogenic” model [1], where the standard model (SM) of quarks and leptons is augmented with a second scalar doublet (η^+, η^0) and three neutral singlet fermions N_R as shown in Fig. 1. Under an exactly

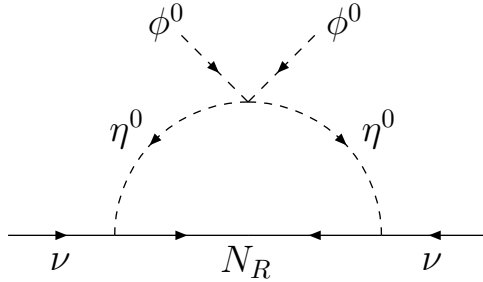


Figure 1: One-loop “scotogenic” neutrino mass.

conserved discrete Z_2 symmetry, (η^+, η^0) and N are odd, allowing thus the existence of dark matter (DM). Whereas such models are viable phenomenologically, a deeper theoretical understanding of the origin of this connection is clearly desirable.

Another important input to this framework is the 2012 discovery of the 125 GeV particle [2, 3] at the Large Hadron Collider (LHC) which looks very much like the one Higgs boson of the SM. This means that any extension of the SM should aim for a natural explanation of why electroweak symmetry breaking appears to be embodied completely in one Higgs scalar doublet and no more.

To these ends, we propose in this paper an anomaly-free $U(1)_X$ gauge extension of the SM with three active and one sterile neutrinos. Whereas there exist many studies on light sterile neutrino masses [4, 5], we consider here for the first time the case where all masses and mixing of active and sterile neutrinos are generated in one loop through dark matter, which is stabilized by a residual Z_2 symmetry of the spontaneously broken $U(1)_X$ gauge symmetry. To maintain the hypothesis of only one electroweak symmetry breaking Higgs

doublet (which couples directly only to quarks in this model), charged-lepton masses are also radiatively generated through dark matter.

The $U(1)_X$ gauge symmetry being considered is a variation of Model (C) of Ref. [6]. It has its origin from the observation [7, 8, 9, 10] that replacing the neutral singlet fermion N of the Type I seesaw for neutrino mass with the fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ of the Type III seesaw also results in a possible $U(1)$ gauge extension. The former is the well-known $B - L$, the latter is the model of Ref. [8], where there is one Σ for each of the three families of quarks and leptons. Here we consider a total of only two Σ 's, in which case several N 's of different $U(1)_X$ charges must be added to render the model anomaly-free. Model (C) of Ref. [6] is the first such example with three N 's. It allows radiative neutrino masses with dark matter [11, 12, 13]. It may also accommodate a sterile neutrino with radiative mass [5], but then dark matter is lost. Here we choose to satisfy the anomaly-free conditions with three different N 's. In so doing, we obtain a model with dark matter as well as radiative masses and mixing for three active and one sterile neutrinos as described below.

Under $U(1)_X$, let three families of $(u, d)_L$, u_R , d_R , $(\nu, e)_L$, e_R transform as $n_{1,2,3,4,5}$ respectively. We add two copies of $(\Sigma^+, \Sigma^0, \Sigma^-)$, each transforming as n_6 . As shown in Ref. [6], the conditions for the absence of axial-vector anomalies in the presence of $U(1)_X$ determine $n_{2,3,5,6}$ in terms of n_1 and n_4 with $3n_1 + n_4 \neq 0$. To satisfy the $\sum U(1)_X^3 = 0$ condition and the $\sum U(1)_X = 0$ condition due to the mixed gravitational-gauge anomaly, three neutral singlet N 's are added. In Model (C) of Ref. [6], their charges [in units of $(3n_1 + n_4)/8$] are $(3, 2, -5)$. Here we choose instead $(-6, 1, 5)$. Note that

$$3^3 + 2^3 + (-5)^3 = -90, \quad 3 + 2 - 5 = 0, \quad (1)$$

$$(-6)^3 + 1^3 + 5^3 = -90, \quad -6 + 1 + 5 = 0, \quad (2)$$

i.e. they give identical contributions to the anomaly-free conditions. However, the latter choice leads to the new model with particle content given in Table 1. The various scalars

particle	a_1	a_4	Z_2
$(u, d)_L$	1	0	+
u_R	7/4	-3/4	+
d_R	1/4	3/4	+
$(\nu, l)_L$	0	1	+
l_R	-9/4	5/4	+
$\Sigma_{1R,2R}^{(+,0,-)}$	9/8	3/8	-
N_R	-9/4	-3/4	+
S_{1R}	3/8	1/8	-
S_{2R}	15/8	5/8	-
$\phi^{(+,0)}$	3/4	-3/4	+
$\eta_1^{(+,0)}$	3/8	-7/8	-
$\eta_2^{(+,0)}$	9/8	-5/8	-
χ_1^0	3/4	1/4	+
χ_2^0	9/4	3/4	+
χ_3^0	3/8	1/8	-
χ_4^+	3/8	-15/8	-
$\xi^{(++ ,+,0)}$	9/8	-13/8	-

Table 1: Particle content of proposed model with $U(1)_X$ assignment given by $a_1 n_1 + a_4 n_4$ where $3n_1 + n_4 \neq 0$.

have been added to allow for all fermions to acquire nonzero masses. An automatic residual Z_2 symmetry is obtained as $U(1)_X$ is spontaneously broken by $\chi_{1,2}^0$. The three neutral singlet fermions are relabelled N_R and $S_{1R,2R}$.

The two heavy fermion triplets obtain masses from the $\Sigma_R \Sigma_R \bar{\chi}_2^0$ interactions, whereas $S_{1R,2R}$ do so through $S_{1R} S_{2R} \bar{\chi}_2^0$ and $S_{1R} S_{1R} \bar{\chi}_1^0$. The quarks get tree-level masses from $\bar{u}_R(u_L \phi^0 - d_L \phi^+)$ and $(\bar{u}_L \phi^+ + \bar{d}_L \phi^0) d_R$. Note that Φ is the only scalar doublet with even Z_2 , corresponding to the one Higgs doublet of the SM, solely responsible for electroweak symmetry breaking. The three active neutrinos ν_L and the one singlet “sterile” neutrino N_R

are massless at tree level. They acquire radiative masses in one loop as shown in Figs. 2 to 4. The requisite couplings are $\bar{\Sigma}_R^0 \nu_L \eta_2^0$, $\Phi^\dagger \eta_2 \bar{\chi}_3^0$, $\chi_3 \chi_3 \bar{\chi}_1^0$, $\bar{S}_{1R} \nu_L \eta_1^0$, $\Phi^\dagger \eta_1 \chi_3^0$, and $N_R S_{2R} \chi_3^0$.

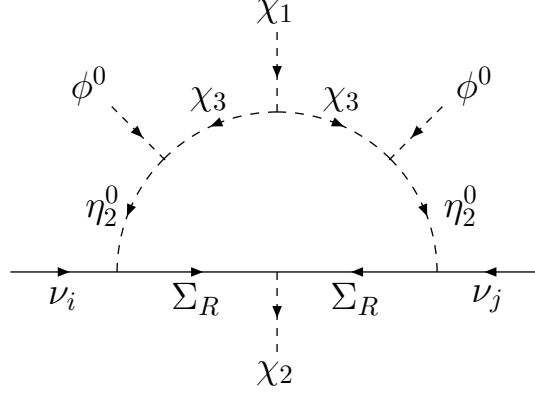


Figure 2: One-loop active neutrino mass from Σ .

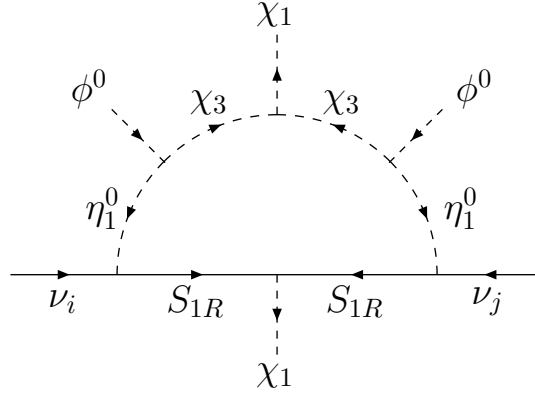


Figure 3: One-loop active neutrino mass from S .

Looking at the one-loop diagrams of Figs. 2 and 3, we see that instead of just one extra scalar doublet η in the original scotogenic model [1], we now have two: one to couple to the two Σ 's, the other to S_{1R} . More importantly, because of the $U(1)_X$ assignments of Σ_R and S_R which come from the anomaly-free conditions, they are odd under the unbroken residual Z_2 allowing the existence of dark matter. At the same time, the neutral singlet fermion N_R is even under Z_2 and massless at tree level, so it is suitable as a light sterile neutrino

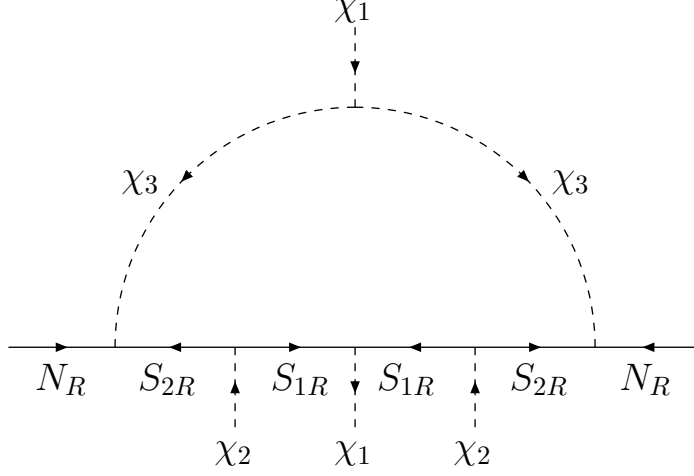


Figure 4: One-loop sterile neutrino mass from S .

once it acquires a radiative mass through S . Thus this anomaly-free $U(1)_X$ model naturally accommodates three active neutrinos and one sterile neutrino, all of which obtain radiative masses. Note that the quartic couplings $\Phi^\dagger \eta_2 \bar{\chi}_1^0 \chi_3^0$ and $\Phi^\dagger \eta_1 \bar{\chi}_3^0 \chi_1^0$ are allowed, which also contribute to Figs. 2 and 3 respectively.

To evaluate the one-loop diagrams of Figs. 1 to 4, we note first that each is a sum of simple diagrams with one internal fermion line and one internal scalar line. Each contribution is infinite, but the sum is finite. In Fig. 1, it is given by [1]

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} [F(m_R^2/M_k^2) - F(m_I^2/M_k^2)], \quad (3)$$

where M_k ($k = 1, 2, 3$) are the three N_R Majorana masses, m_R is the $\sqrt{2}Re(\eta^0)$ mass, m_I is the $\sqrt{2}Im(\eta^0)$ mass, and $F(x) = x \ln x / (x - 1)$. In Figs. 2 to 4, we need to consider the more complicated scalar and fermion sectors. There are 8 real scalar fields, spanning $\sqrt{2}Re(\eta_{1,2}^0)$, $\sqrt{2}Im(\eta_{1,2}^0)$, $\sqrt{2}Re(\chi_3^0)$, $\sqrt{2}Im(\chi_3^0)$, $\sqrt{2}Re(\xi^0)$, $\sqrt{2}Im(\xi^0)$. Let their mass eigenstates be ζ_l with mass m_l . There are 4 Majorana fermion fields, spanning Σ_{1R}^0 , Σ_{2R}^0 , S_{1R} , S_{2R} . Let their mass eigenstates be ψ_k with mass M_k .

In Fig. 2, let the $\bar{\Sigma}_{1R}^0 \nu_i \eta_2^0$ and $\bar{\Sigma}_{2R}^0 \nu_i \eta_2^0$ couplings be $h_{i1}^{(2)}$ and $h_{i2}^{(2)}$, then its contribution to \mathcal{M}_ν is given by

$$(\mathcal{M}_\nu)_{ij}^{(2)} = \frac{h_{i1}^{(2)} h_{j1}^{(2)}}{16\pi^2} \sum_k (z_{1k}^\Sigma)^2 M_k \sum_l [(y_{2l}^R)^2 F(x_{lk}) - (y_{2l}^I)^2 F(x_{lk})] \\ + \frac{h_{i2}^{(2)} h_{j2}^{(2)}}{16\pi^2} \sum_k (z_{2k}^\Sigma)^2 M_k \sum_l [(y_{2l}^R)^2 F(x_{lk}) - (y_{2l}^I)^2 F(x_{lk})], \quad (4)$$

where $\Sigma_{1R}^0 = \sum_k z_{1k}^\Sigma \psi_k$, $\Sigma_{2R}^0 = \sum_k z_{2k}^\Sigma \psi_k$, $\sqrt{2}Re(\eta_2^0) = \sum_l y_{2l}^R \zeta_l$, $\sqrt{2}Im(\eta_2^0) = \sum_l y_{2l}^I \zeta_l$, with $\sum_k (z_{1k}^\Sigma)^2 = \sum_k (z_{2k}^\Sigma)^2 = \sum_l (y_{2l}^R)^2 = \sum_l (y_{2l}^I)^2 = 1$, and $x_{lk} = m_l^2/M_k^2$. In Fig. 3, let the $\bar{S}_{1R} \nu_i \eta_1^0$ coupling be $h_{i1}^{(1)}$, then its contribution to \mathcal{M}_ν is given by

$$(\mathcal{M}_\nu)_{ij}^{(1)} = \frac{h_{i1}^{(1)} h_{j1}^{(1)}}{16\pi^2} \sum_k (z_{1k}^S)^2 M_k \sum_l [(y_{1l}^R)^2 F(x_{lk}) - (y_{1l}^I)^2 F(x_{lk})], \quad (5)$$

where $S_{1R} = \sum_k z_{1k}^S \psi_k$, $\sqrt{2}Re(\eta_1^0) = \sum_l y_{1l}^R \zeta_l$, $\sqrt{2}Im(\eta_1^0) = \sum_l y_{1l}^I \zeta_l$, with $\sum_k (z_{1k}^S)^2 = \sum_l (y_{1l}^R)^2 = \sum_l (y_{1l}^I)^2 = 1$. In Fig. 4, let the $S_{2R} N_R \chi_3^0$ coupling be $h_2^{(3)}$, then

$$m_N = \frac{h_2^{(3)} h_2^{(3)}}{16\pi^2} \sum_k (z_{2k}^S)^2 M_k \sum_l [(y_{3l}^R)^2 F(x_{lk}) - (y_{3l}^I)^2 F(x_{lk})], \quad (6)$$

where $S_{2R} = \sum_k z_{2k}^S \psi_k$, $\sqrt{2}Re(\chi_3^0) = \sum_l y_{3l}^R \zeta_l$, $\sqrt{2}Im(\chi_3^0) = \sum_l y_{3l}^I \zeta_l$, with $\sum_k (z_{2k}^S)^2 = \sum_l (y_{3l}^R)^2 = \sum_l (y_{3l}^I)^2 = 1$.

In the above, the three active neutrinos $\nu_{1,2,3}$ acquire masses through their couplings to three dark neutral fermions, i.e. Σ_{1R}^0 , Σ_{2R}^0 , S_{1R} , whereas the one sterile neutrino N acquires mass through its coupling to S_{2R} . However, since S_{1R} mixes with S_{2R} at tree level, there is also mixing between ν_i and N as shown in Fig. 5, with

$$m_{\nu N} = \frac{h_{i1}^{(1)} (h_2^{(3)})^*}{16\pi^2} \sum_k z_{1k}^S z_{2k}^S M_k \sum_l [y_{1l}^R y_{3l}^R F(x_{lk}) - y_{1l}^I y_{3l}^I F(x_{lk})], \quad (7)$$

where $\sum_k z_{1k}^S z_{2k}^S = \sum_l y_{1l}^R y_{3l}^R = \sum_l y_{1l}^I y_{3l}^I = 0$. Note that the structures of these one-loop formulas are all similar, and there is enough freedom in choosing the various parameters to obtain masses of order 0.1 eV for ν and 1 eV for N , as well as a sizeable $\nu - N$ mixing. Note

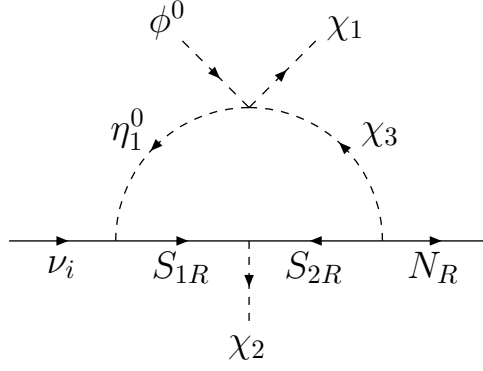


Figure 5: One-loop active-sterile neutrino mixing from S .

also that the last term in each case corresponds to the cancellation among several scalars which allow the loops to be finite and should be naturally small. In Fig. 1, it is represented by the well-known $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$ term which splits $Re(\eta^0)$ and $Im(\eta^0)$ in mass. In our case for example, in Eqs. (4) and (5), let $h \sim 10^{-1}$, the $\sum z^2 M$ factor ~ 1 TeV, the $\sum[(y^R)^2 - (y^I)^2]F$ factor $\sim 10^{-9}$ (which means that the $\bar{\chi}_1\chi_3^2$ coupling is very small), then $m_\nu \sim 0.1$ eV. In Eq. (6), let the $\sum[(y^R)^2 - (y^I)^2]F$ factor $\sim 10^{-8}$ instead, then $m_N \sim 1$ eV. In Eq. (7), let the $\sum z_1 z_2 M$ factor be 100 GeV, and the $\sum[y^R y^R - y^I y^I]F$ factor $\sim 10^{-9}$ (which means that the $\eta_1^\dagger \Phi \bar{\chi}_1 \chi_3$ coupling is very small), then $m_{\nu N} \sim 0.1$ eV. This is thus a possible framework for accommodating three active neutrinos plus a fourth light sterile neutrino, in the 3+1 scheme [14], with best fit values $\Delta m_{41}^2 = 0.93$ eV², $|U_{e4}| = 0.15$, $|U_{\mu 4}| = 0.17$, albeit having a large χ^2 , to ease the long-standing tension between ν_e appearance and ν_μ disappearance experiments in the $\Delta m^2 \sim \text{few eV}^2$ range.

Our model is also an example of the recently proposed framework [15, 16], where charged leptons also acquire radiative masses. Indeed, they do so here also through the same four dark fermions, i.e. Σ_{1R}^0 , Σ_{2R}^0 , S_{1R} , S_{2R} , with the addition of a scalar triplet $\xi^{(+++,0)}$ and a scalar singlet χ_4^+ , as shown in Figs. 6 and 7. There are 4 charged scalars, spanning η_1^+ , η_2^+ , ξ^+ , χ_4^+ . Let their mass eigenstates be ω_r^+ with mass m_r . In Fig. 6, let the $l_{jR}\Sigma_{1R}\xi^+$ and

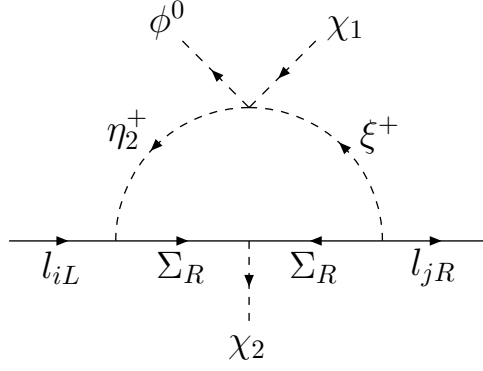


Figure 6: One-loop charged-lepton mass from Σ .

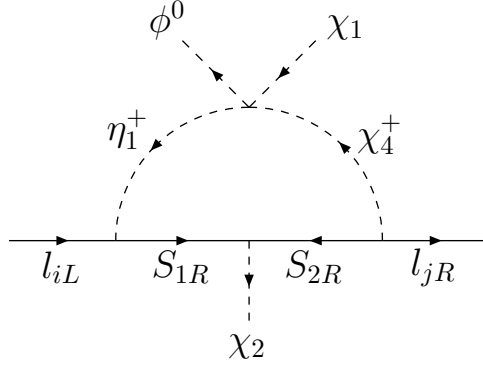


Figure 7: One-loop charged-lepton mass from S .

$l_{jR}\Sigma_{2R}\xi^+$ couplings be h_{j1}^ξ and h_{j2}^ξ , then its contribution to \mathcal{M}_l is

$$\begin{aligned}
 (\mathcal{M}_l)_{ij}^{(\xi)} &= \frac{h_{i1}^{(2)}(h_{j1}^\xi)^*}{16\pi^2} \sum_k (z_{1k}^\Sigma)^2 M_k \sum_r y_{2r}^+ y_{\xi r} F(x_{rk}) \\
 &+ \frac{h_{i2}^{(2)}(h_{j2}^\xi)^*}{16\pi^2} \sum_k (z_{2k}^\Sigma)^2 M_k \sum_r y_{2r}^+ y_{\xi r} F(x_{rk}),
 \end{aligned} \tag{8}$$

where $\eta_2^+ = \sum_r y_{2r}^+ \omega_r^+$, $\xi^+ = \sum_r y_{\xi r} \omega_r^+$. In Fig. 7, let the $l_{jR}S_{2R}\chi_4^+$ coupling be h_j^χ , then its contribution to \mathcal{M}_l is

$$(\mathcal{M}_l)_{ij}^{(\chi)} = \frac{h_{i1}^{(2)}(h_j^\chi)^*}{16\pi^2} \sum_k z_{1k}^S z_{2k}^S M_k \sum_r y_{1r}^+ y_{\chi r} F(x_{rk}), \tag{9}$$

where $\eta_1^+ = \sum_r y_{1r}^+ \omega_r^+$, $\chi_4^+ = \sum_r y_{\chi r} \omega_r^+$. Let the $\sum y^+ y F$ factor ~ 1 , and vary $h^{\xi,\chi}$ from 1 to 0.1 to 0.001, then m_τ, m_μ, m_e may be obtained. Here we require the $\eta_2 \Phi \bar{\chi}_1 \xi^\dagger$ and $\eta_1 \Phi \bar{\chi}_1 \chi_4$

couplings be large. One immediate consequence of radiative charged-lepton masses is the possible significant deviation of the Higgs Yukawa coupling to $\bar{l}l$ from the value $m_l/(246 \text{ GeV})$ required by the SM. Detailed analyses [17, 18] have been performed for some specific models.

The neutral dark scalars ζ_l have in general components which are not electroweak singlets ($\eta_{1,2}^0, \xi^0$). As such, they are not good dark-matter candidates because their interactions with the Z gauge boson would result in too large a cross section for their direct detection in underground experiments. Hence one of the neutral dark fermions ψ_k is a much better DM candidate. Note that whereas Σ_{1R}^0 and Σ_{2R}^0 are components of $SU(2)_L$ triplets, they do not couple to Z because they have $I_3 = 0$. Note also that they mix with S_{1R} and S_{2R} only in one loop. The case of Σ^0 as dark matter in the triplet fermion analog of the scotogenic model was discussed in Ref. [19]. Here the important change is that Σ_{1R}^0 and Σ_{2R}^0 have both $SU(2)_L$ and $U(1)_X$ interactions. On the other hand, suppose the lighter linear combination of S_{1R} and S_{2R} is dark matter, call it ψ_0 , then only $U(1)_X$ is involved. As shown recently in [12, 13], the allowed region of parameter space from dark matter relic abundance, direct detection and collider constraints corresponds to the s-wave resonance region near $m_X \approx 2m_{\psi_0}$. The $U(1)_X$ gauge boson mass m_X in our model comes from $\langle \chi_{1,2} \rangle = u_{1,2}$. If $n_1 = n_4 = 1$ is chosen in Table 1, then the SM Higgs does not transform under $U(1)_X$ and there is no $X - Z$ mixing. In that case,

$$m_X^2 = 2g_X^2(u_1^2 + 9u_2^2). \quad (10)$$

The $U(1)_X$ charges of $(u, d)_L$, u_R , d_R , $(\nu, l)_L$ are all 1, and those of l_R , N_R , χ_1 , χ_2 are -1 , -3 , 1 , 3 . These particles are even under the residual Z_2 of $U(1)_X$. The dark sector consists of fermions Σ_{1R} , Σ_{2R} , S_{1R} , S_{2R} , with $U(1)_X$ charges $3/2$, $3/2$, $1/2$, $5/2$, as well as scalars η_1 , η_2 , χ_3 , χ_4 , ξ , with $U(1)_X$ charges $-1/2$, $1/2$, $1/2$, $-3/2$, $-1/2$.

Instead of having a sterile neutrino of 1 eV, it is also possible in our model to make it a

few keV, thus rendering N a warm dark-matter candidate. This may require $h_2^{(3)}$ in Eq. (6) to be much greater than the corresponding Yukawa couplings in Eqs. (4) and (5) for the active neutrinos. On the other hand, $\nu - N$ mixing has to be much more suppressed in order not to overclose the Universe or conflict with observed X-ray data. According to Ref. [20], these may be avoided if the mixing $|U_{i4}|$ is less than 10^{-4} . Such a small mixing also makes the keV sterile neutrino long-lived on cosmological time scales. It could also provide an explanation to the recently observed 3.55 keV X-ray line [21] after analysing the data taken by the XMM-Newton X-Ray telescope in the spectrum of 73 galaxy clusters. The same line also appears in the *Chandra* observations of the Perseus cluster [22] and the XMM-Newton observations of the Milky Way Centre [23]. In the absence of any astrophysical interpretation of the line due to some atomic transitions, the origin of this X-ray line can be explained naturally by sterile neutrino dark matter with mass approximately 7.1 keV decaying into a photon and a standard model neutrino. As reported in Ref. [22], the required mixing angle of sterile neutrino with active neutrino should be of the order of $\sin^2 2\theta \approx 10^{-11} - 10^{-10}$ in order to give rise to the observed X-Ray line flux. For such a tiny mixing angle, Fig. 5 must be strongly suppressed, implying thus almost zero $\chi_3 - \eta_1$ mixing. This in turn will make Fig. 3 vanish, thus predicting one nearly massless active neutrino.

We have shown in this paper how a stabilizing Z_2 symmetry for dark matter may be derived from a new anomaly-free $U(1)_X$ extension of the standard model. Using just the one Higgs doublet of the SM, we have also shown how three charged leptons and active neutrinos plus a sterile neutrino acquire radiative masses through the dark sector. This explains why the sterile neutrino mass itself is also small. Apart from the possibilities of long lived sterile neutrino dark matter and cold dark matter separately as discussed above, our model is well-suited for the much more interesting mixed-dark-matter scenario, i.e. the coexistence of both. Such a scenario could be important from the point of view of large structure formation, as

well as offering proofs in different indirect detection experiments ranging from gamma rays to X-rays. We leave such a complete analysis to future investigations.

This work is supported in part by the U. S. Department of Energy under Grant No. de-sc0008541.

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